$$\int_1^4 x^4 \, dx$$

Midpoint Rule n=6

i	$\bar{x}_i$	$f(\bar{x}_i)$
1	1.25	2.44140625
2	1.75	9.37890625
3	2.25	25.62890625
4	2.75	57.19140625
5	3.25	111.56640625
6	3.75	197.75390625
	TOTAL	**403.9609375**

$$M_n = \Delta x\left[\sum_{i=1}^{n} f\left(\bar{x}_i\right)\right] = (.5)(403.9609375) = 201.98046875$$

$$E_M = 201.98046875 - 204.6 = -2.61953125$$

$$\int_1^4 x^4 \, dx$$

Trapezoid Rule n=6

i	x_i	$f(x_i)$	w_i=weighting	$w_i \cdot f(x_i)$
0	1	1	1	1
1	1.5	5.0625	2	10.125
2	2	16	2	32
3	2.5	39.0625	2	78.125
4	3	81	2	162
5	3.5	150.0625	2	300.125
6	4	256	1	256
			TOTAL	**839.375**

$$T_n = \Delta x \left[\frac{1}{2} \sum_{i=0}^{n} w_i f\left(x_i\right) \right] = (.5)(.5)(839.375) = 209.84375$$

$$E_T = 209.84375 - 204.6 = +5.24375$$

Estimate $\int_{1}^{2.2} f(x)\,dx$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	1	0.5	-0.2	-0.1	0.4	1.1	2.0

Trapezoid Rule

i	x_i	$f(x_i)$	w_i	$w_i \cdot f(x_i)$
0	1.0	1.0	1	1.0
1	1.2	0.5	2	1.0
2	1.4	-0.2	2	-0.4
3	1.6	-0.1	2	-0.2
4	1.8	0.4	2	0.8
5	2.0	1.1	2	2.2
6	2.2	2.0	1	2.0
			Total	**6.4**

$$T_n = \Delta x\left[\frac{1}{2}\sum_{i=1}^{n} w_i\, f\left(x_i\right)\right] = (.2)(.5)(6.4) = 0.64$$

Estimate $\int_{1}^{2.2} f(x)\,dx$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	1	0.5	-0.2	-0.1	0.4	1.1	2.0

Midpoint Rule

i	x_i	$f(x_i)$	$\bar{x}_i$	$f(\bar{x}_i)$
0	1.0	1.0		
1	1.2	0.5	1.1	0.75
2	1.4	-0.2	1.3	0.15
3	1.6	-0.1	1.5	-0.15
4	1.8	0.4	1.7	0.15
5	2.0	1.1	1.9	0.75
6	2.2	2.0	2.1	1.55
			Total	**3.20**

$$M_n = \Delta x\left[\sum_{i=1}^{n} f\left(\bar{x}_i\right)\right] = (.2)(3.2) = 0.64$$

$$\int_{1}^{2.2} f(x)\,dx$$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	1	0.5	-0.2	-0.1	0.4	1.1	2.0

$$\int_a^b e^{-x^2}\,dx$$

ERROR BOUND - TRAPEZOID RULE

$\left\|E_T\right\| \le \dfrac{K(b-a)^3}{12n^2}$	n= number of sub-intervals B = upper bound A = lower bound K=largest value of $\left\|f''(x)\right\|$ over (a,b)

ERROR BOUND - MIDPOINT RULE

$\left\|E_M\right\| \le \dfrac{K(b-a)^3}{24n^2}$	n= number of sub-intervals B = upper bound A = lower bound K=largest value of $\left\|f''(x)\right\|$ over (a,b)

ERROR BOUND – SIMPSON'S RULE

$\left\|E_S\right\| \le \dfrac{K(b-a)^5}{180n^4}$	n= number of sub-intervals B = upper bound A = lower bound K=largest value of $\left\|f^{(4)}(x)\right\|$ over (a,b)

$$\int_1^4 x^3 \, dx$$

Simpsons Rule n=12

i	x_i	$f(x_i)$	w_i	$w_i \cdot f(x_i)$
0	1	1.00000000	1	1.000000
1	1.25	2.44140625	4	9.765625
2	1.50	5.06250000	2	10.125000
3	1.75	9.37890625	4	37.515625
4	2.00	16.00000000	2	32.000000
5	2.25	25.62890625	4	102.515625
6	2.50	39.06250000	2	78.125000
7	2.75	57.19140625	4	228.765625
8	3.00	81.00000000	2	162.000000
9	3.25	111.56640625	4	446.265625
10	3.50	150.06250000	2	300.125000
11	3.75	197.75390625	4	791.015625
12	4.00	256.00000000	1	256.000000
			TOTAL	**2455.218750**

$$S_n = \Delta x \left[\frac{1}{3} \sum_{i=0}^{n} w_i \, f\left(x_i\right) \right] = \frac{1}{12}(2455.218750) = 204.6015625$$

$$E_S = 204.6015625 - 204.6 = 0.0015625$$

$$\int_{1}^{2.2} f(x)\,dx$$

SIMPSON'S RULE

i	x_i	$f(x_i)$	w_i	$w_i \cdot f(x_i)$
0	1.0	1.0	1	1.0
1	1.2	0.5	4	2.0
2	1.4	-0.2	2	-0.4
3	1.6	-0.1	4	-0.4
4	1.8	0.4	2	0.8
5	2.0	1.1	4	4.4
6	2.2	2.0	1	2.0
			Total	**9.4**

$$S_n = \Delta x\left[\frac{1}{3}\sum_{i=0}^{n} w_i\, f\left(x_i\right)\right] = \frac{1}{3}(.2)(9.4) = 0.6267$$